

ν DFSZ: a technically natural non-supersymmetric model of neutrino masses, baryogenesis, the strong CP problem, and dark matter

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We describe a minimal extension of the standard model by three right-handed neutrinos, a scalar doublet, and a scalar singlet (the “ ν DFSZ”) which serves as an existence proof that weakly coupled high-scale physics can naturally explain phenomenological shortcomings of the SM. The ν DFSZ can explain neutrino masses, baryogenesis, the strong CP problem, and dark matter, and remains calculably natural despite a hierarchy of scales up to $\sim 10^{11}$ GeV. It predicts a SM-like Higgs boson, (maximally) TeV-scale scalar states, intermediate-scale hierarchical leptogenesis (10^5 GeV $\lesssim M_N \lesssim 10^7$ GeV), and axionic dark matter.

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I. Introduction: The standard model (SM) and the paradigm of spontaneous electroweak symmetry breaking, realised by a scalar potential

$$V_{SM} = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (1)$$

where $\mu^2(m_Z) \approx -(88 \text{ GeV})^2$, has proven extremely successful in explaining low energy phenomena. Nevertheless, it fails to explain neutrino masses, the baryon asymmetry of the Universe (BAU), the smallness of the neutron electric dipole moment (the strong CP problem), dark matter, and gravity. Whether nature realises these phenomena in a “natural” way, i.e., in such a way that μ^2 is (sufficiently) insensitive to physically meaningful quantum corrections, remains an open question. Still, motivated by aesthetics, the pursuit of a natural “theory of everything” has motivated much of modern particle physics.

In the same vein, this paper describes an extension of the SM by three right-handed neutrinos, a scalar doublet, and a scalar singlet. The model can be thought of as an extension of the Dine–Fischler–Srednicki–Zhitnitsky (DFSZ) invisible axion model [1, 2] by right-handed neutrinos, and is thus henceforth referred to as the ν DFSZ. Notably, as we will describe in this paper, the ν DFSZ can explain neutrino masses, the BAU, the strong CP problem, and dark matter *in a calculably natural way*, even despite a hierarchy of scales up to $\sim 10^{11}$ GeV. This is achieved by a seesaw mechanism, hierarchical leptogenesis, the Peccei–Quinn (PQ) mechanism, an invisible axion, and a technically natural decoupling limit, respectively.

The paper is organised as follows. We first detail the ν DFSZ, its vacuum, and its scalar sector (and constraints). We then describe how it provides explanations for the strong CP problem, dark matter, neutrino masses, and the BAU. Penultimately, we describe our naturalness philosophy, identify the symmetries which protect each scale from quantum corrections, and study an example point in the parameter space. Finally we conclude.

II. The ν DFSZ Lagrangian: The scalar content of the model is a complex singlet S and two complex doublets $\Phi_{1,2}$ of hypercharge +1. The potential is¹

$$\begin{aligned} V_{\nu\text{DFSZ}} = & M_{11}^2 \Phi_1^\dagger \Phi_1 + M_{22}^2 \Phi_2^\dagger \Phi_2 + M_{SS}^2 S^* S \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \frac{\lambda_S}{2} (S^* S)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ & + \lambda_{1S} (\Phi_1^\dagger \Phi_1) (S^* S) + \lambda_{2S} (\Phi_2^\dagger \Phi_2) (S^* S) \\ & - \epsilon \Phi_1^\dagger \Phi_2 S^2 - \epsilon \Phi_2^\dagger \Phi_1 S^{*2}, \end{aligned} \quad (2)$$

where $M_{SS}^2 \sim -(10^{11} \text{ GeV})^2 \equiv -M_{PQ}^2$ sets the PQ scale. Additional terms otherwise allowed by gauge symmetry are forbidden by a global $U(1)_{PQ}$ symmetry to be defined in Sec. V, which is essential in solving the strong CP problem. The ϵ terms are necessary² to assign a PQ charge to S and help to generate neutrino masses once S obtains a vacuum expectation value (vev).

The only addition to the SM fermionic content is three right-handed neutrinos. The strong CP solution dictates that Φ_1 (Φ_2) couple to u_R (d_R), and our solution for natural neutrino masses and leptogenesis requires that Φ_2 couple to ν_R . The Yukawa Lagrangian is therefore

$$\begin{aligned} -\mathcal{L}_Y = & y_u \bar{q}_L \tilde{\Phi}_1 u_R + y_d \bar{q}_L \Phi_2 d_R \\ & + y_e \bar{l}_L \Phi_J e_R + y_\nu \bar{l}_L \tilde{\Phi}_2 \nu_R \\ & + \frac{1}{2} y_N (\nu_R)^c S \nu_R + H.c., \end{aligned} \quad (3)$$

where family indices are implied, $\tilde{\Phi}_i \equiv i\tau_2 \Phi_i^*$, and $J = 2$ (1) is a Type II (Flipped) ν -two-Higgs-doublet model

¹ As far as we are aware, the ν DFSZ was first discussed in Ref. [3]. See Refs. [4–6] and references therein for other minimal approaches to connecting the PQ mechanism with neutrino masses.

² Another option is to have terms $-(\kappa \Phi_1^\dagger \Phi_2 S + H.c.)$ [7].

($\nu 2\text{HDM}$) arrangement. We will work in the basis where y_N is diagonal and real. Again, additional terms are forbidden by the $U(1)_{PQ}$ symmetry.³

We note here that each of $\epsilon \rightarrow 0$, $y_N \rightarrow 0$, and $y_\nu \rightarrow 0$ is a technically natural limit, since they lead to an extra $U(1)$ symmetry which can be identified with lepton number. As well, there are two apparent technically natural decoupling limits associated with enhanced Poincaré symmetries [8]: $\epsilon, \lambda_{1S}, \lambda_{2S}, y_N \rightarrow 0$ decouples S , and $\epsilon, \lambda_{1S}, \lambda_{2S}, y_\nu \rightarrow 0$ decouples the (ν_R, S) subsystem. These limits will prove important in protecting the hierarchy of scales in the model.

III. Vacuum: The scalar fields acquire vevs $\langle S \rangle \equiv v_S/\sqrt{2}$, $\langle \Phi_i \rangle \equiv (0, v_i/\sqrt{2})^T$. If $v_S \gg v_i$, then

$$v_S \approx \sqrt{\frac{-2M_{SS}^2}{\lambda_S}} \sim 10^{11} \text{ GeV}. \quad (4)$$

Expanding around this vev, the right-handed neutrinos acquire Majorana masses $M_N = y_N \langle S \rangle$ and the scalar potential becomes

$$V_{\nu 2\text{HDM}} \approx m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \dots, \quad (5)$$

where $m_{ii}^2 = M_{ii}^2 + \lambda_{iS} \langle S \rangle^2$ and $m_{12}^2 = \epsilon \langle S \rangle^2$.

We will adopt the natural explanation of neutrino masses and baryogenesis detailed in Ref. [9]. This requires $v_2 \sim \mathcal{O}(1-10)$ GeV achieved with $m_{11}^2 < 0$, $m_{22}^2 > 0$, and $m_{12}^2/m_{22}^2 \ll 1$.⁴ Anticipating $m_{22}^2 \gg v_1^2(\lambda_3 + \lambda_4)$, $\lambda_2 v_2^2$, the Φ_i vevs can be written

$$v_2 \equiv \frac{v_1}{\tan \beta} \approx \frac{m_{12}^2}{m_{22}^2} v_1, \quad v_1 \approx \sqrt{\frac{2}{\lambda_1} \left(-m_{11}^2 + \frac{m_{22}^2}{\tan^2 \beta} \right)}, \quad (6)$$

where $\sqrt{v_1^2 + v_2^2} = v \approx 246$ GeV and we have implicitly defined $\tan \beta$. There is an important consistency condition $2m_{22}^2/\tan^2 \beta \lesssim \lambda_1 v_1^2$ to ensure $m_{11}^2 < 0$ and avoid a fine-tuning for v (see Ref. [9]).

Typical values for the mass parameters are $m_{11}^2 \approx -(88 \text{ GeV})^2$, $m_{22} \sim 10^3$ GeV, and $m_{22}^2/\tan^2 \beta \ll |m_{11}^2|$. Therefore, for no fine-tuning between M_{ii}^2 and m_{ii}^2 , we already expect $\lambda_{1S} \lesssim 10^{-18}$, $\lambda_{2S} \lesssim 10^{-16}$, and $\epsilon \ll 10^{-18}$.

IV. Scalar sector: The scalar mass eigenstates are, up to v_1/m_{22} and m_{12}^2/m_{22}^2 corrections (see Ref. [9] for expressions), a CP even state (h) with $m_h^2 \approx \lambda_1 v_1^2$, three

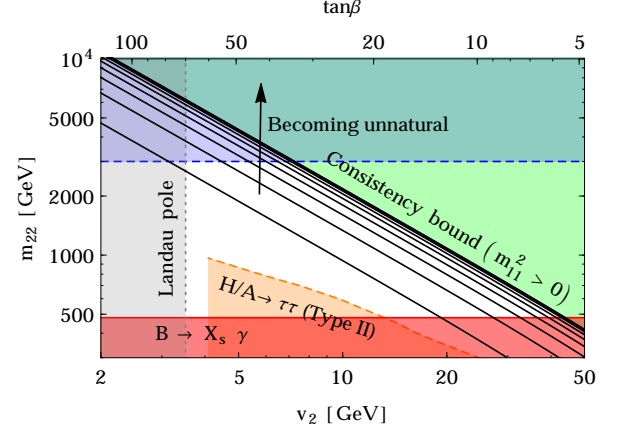


FIG. 1. Constraints on m_{22} , as labelled. Solid black contours are $m_{11}^2/\text{GeV}^2 = -80^2, -70^2$, and so on. The $H/A \rightarrow \tau\tau$ bound is taken from the CMS search [14]. The naturalness bound is only illustrative (see Ref. [9]).

heavy scalar states (H, A, H^\pm) with masses $\approx m_{22}$, a PQ-scale neutral scalar (s) with $m_s^2 = \lambda_S v_S^2$, and a very light pseudo-Goldstone boson (the invisible axion).

Owing to the approximate $U(1)$ symmetry due to $m_{12}^2/m_{22}^2 \ll 1$ and $v^2/m_{22}^2 \ll 1$, the state h closely resembles the SM Higgs.⁵ In Fig. 1 we illustrate the various constraints on m_{22} . These are the aforementioned consistency condition, measurements of $B \rightarrow X_s \gamma$ [11, 12], $H/A \rightarrow \tau\tau$ LHC searches (for the Type II model) [13, 14], perturbativity up to the Planck scale [15], and naturalness [9]. The naturalness bound was derived in Ref. [9] subject to the naturalness condition we describe in Sec. IX, and we refer the reader there for details.

V. The strong CP problem: Gauge invariance and renormalisability permit the physical CP violating term, $\bar{\theta} \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a$ where G is the gluon field strength tensor and \tilde{G} its dual, to be added to the SM Lagrangian. Bounds on the neutron electric dipole moment constrain $\bar{\theta} \lesssim 10^{-10}$ [16]. The strong CP problem is: why is $\bar{\theta} \approx 0$?⁶

The PQ solution [22] to the strong CP problem is to demand a global chiral $U(1)_{PQ}$ symmetry; if the sum of the u_R and d_R PQ charges is non-zero then, after PQ symmetry breaking, $\bar{\theta}$ becomes dynamical, and the vacuum potential “selects” $\langle \bar{\theta} \rangle = 0$. The νDFSZ model is one manifestation of this solution.

Let us now identify the global $U(1)_{PQ}$ symmetry. Defining the $U(1)_{PQ}$ charge names as in Table I, we can (without loss of generality) set $X_q = 0$ and $X_u + X_d = 1$. Equations (2) and (3) set an additional six constraints on

³ In this model the right-handed neutrinos gain mass from the vev of S , but an alternative scenario with explicit Majorana masses is also possible.

⁴ Note that, like $\epsilon \rightarrow 0$ in the original Lagrangian, $m_{12}^2/m_{22}^2 \rightarrow 0$ is a technically natural limit associated with $U(1)_L$ [10].

⁵ Up to v/v_S corrections, at low scale the model is just the $\nu 2\text{HDM}$ discussed in Ref. [9] with a very weakly coupled axion.

⁶ For reviews of the strong CP problem and axions see e.g. Refs. [17–21].

Field	$U(1)_{PQ}$	Value in [Type II, Flipped]
S	X_S	$\frac{1}{2}$
Φ_1	X_1	$\cos^2 \beta$
Φ_2	X_2	$-\sin^2 \beta$
q_L	X_q	0
u_R	X_u	$\cos^2 \beta$
d_R	X_d	$\sin^2 \beta$
l_L	X_l	$\frac{3}{4} - \cos^2 \beta$
ν_R	X_ν	$-\frac{1}{4}$
e_R	X_e	$[\frac{7}{4}, \frac{3}{4}] - 2\cos^2 \beta$

TABLE I. Charges of fields under the PQ symmetry.

the charges, which brings the total to eight for nine unknown charges. They are completely defined by setting $X_1 = \alpha X_2$, as long as $\alpha \neq 1$; it is convenient to choose $\alpha = -\cot^2 \beta$ so that the PQ current does not couple to the field eaten by the Z boson. The resulting charge values are given in Table I.

A final comment before moving on. In the SM, if $\bar{\theta}$ is set to zero by hand at some high scale, renormalisation implies $\bar{\theta} \lesssim 10^{-17}$ [23, 24], well below the experimental bound. In this sense, in the SM, $\bar{\theta} \approx 0$ is *already* natural. Yet this explanation remains unsatisfying, since the limit $\bar{\theta} \rightarrow 0$ cannot be identified with a symmetry. The ν DFSZ solution requires $\lambda_{iS} \ll 1$, and thus one could similarly ask: why are the $\lambda_{iS} \approx 0$? At least, here, this limit is identified with an increased Poincaré symmetry. As well, in the presence of CP violating new physics (such as the right-handed neutrinos), this solution *guarantees* $\bar{\theta} \approx 0$.

VI. Dark matter: The ν DFSZ axion gains a mass

$$m_a \approx 6 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right) \quad (7)$$

due to the chiral anomaly [18, 20], where $f_a \approx \langle S \rangle$ is the axion decay constant, and inherits v/f_a suppressed couplings to nucleons, photons, and electrons (for expressions see e.g. Ref. [18]). Stellar energy loss constrains $f_a \gtrsim 4 \times 10^8 \text{ GeV}$ [18].⁷

The axion provides a cold dark matter candidate via the misalignment mechanism [28–30], wherein a significant amount of energy density is stored in coherent oscillations of the axion field, [19]

$$\Omega_a h^2 \sim 0.7 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{\frac{7}{6}} \left(\frac{\theta}{\pi} \right)^2, \quad (8)$$

where $-\pi \leq \theta \leq \pi$ is the misalignment angle. The requirement that this quantity not exceed the measured

cold dark matter energy density $\Omega_{\text{CDM}} h^2 \approx 0.12$ [31] implies

$$f_a \lesssim 2 \times 10^{11} \text{ GeV} \left(\frac{\pi}{\theta} \right)^{\frac{12}{7}}, \quad (9)$$

with equality reproducing the observed density. If the PQ symmetry is broken after inflation, then the misalignment angle is the average value taken over many distinct patches, $\theta^2 \approx \pi^2/3$, and one obtains $f_a \lesssim 6 \times 10^{11} \text{ GeV}$ [17].⁸ Future projections of the ADMX and CAPP resonant microwave cavity experiments promise to probe this interesting region of parameter space [33, 34].

VII. Neutrino masses: The neutrino mass matrix is given by

$$m_\nu = \frac{v_2^2}{2} y_\nu \mathcal{D}_M^{-1} y_\nu^T \approx \frac{1}{\tan^2 \beta} \left(\frac{v^2}{2} y_\nu \mathcal{D}_M^{-1} y_\nu^T \right), \quad (10)$$

where the bracketed quantity is the typical Type I seesaw formula [35–38]. The mass matrix is diagonalised by a unitary matrix U , $U m_\nu U^T = \text{diag}(m_1, m_2, m_3) \equiv \mathcal{D}_m$, where m_i are the neutrino masses. It will be useful to express y_ν in the Casas–Ibarra [39] form,

$$U y_\nu = \frac{\sqrt{2}}{v_2} \mathcal{D}_m^{\frac{1}{2}} R \mathcal{D}_M^{\frac{1}{2}}, \quad (11)$$

where R is a (possibly complex) orthogonal matrix.

VIII. The BAU: The BAU is produced analogously to standard hierarchical thermal leptogenesis [40], via the out-of-equilibrium, CP violating decays of the lightest right-handed neutrino: $N_1 \rightarrow l \Phi_2$ [9].

If only decays and inverse decays are considered, the evolution of the asymmetry is characterised (in the one-flavour approximation) by the decay parameter K ,

$$K = \frac{\Gamma_D}{H|_{T=M_1}} \equiv \frac{\tilde{m}_1}{m_*}, \quad (12)$$

where Γ_D is the N_1 decay rate, H is the Hubble rate, \tilde{m}_1 is the effective neutrino mass, and m_* is the equilibrium neutrino mass,

$$\Gamma_D = \frac{1}{8\pi} (y_\nu^\dagger y_\nu)_{11} M_1, \quad \tilde{m}_1 \equiv \frac{(y_\nu^\dagger y_\nu)_{11} v_2^2}{2M_1}, \quad (13)$$

$$H \approx \frac{17T^2}{M_{Pl}}, \quad m_* \approx \frac{10^{-3} \text{ eV}}{\tan^2 \beta}. \quad (14)$$

The salient $\Delta L = 1$ scatterings are electroweak, and those involving b quarks and (in Type II) τ leptons. Since those rates scale with the decays and inverse decays, proportional to $(y_\nu^\dagger y_\nu)_{11}$, they can only represent a minor correction to the standard case with electroweak and t

⁷ In a Type II ν DFSZ, red giants and white dwarfs constrain $f_a \gtrsim 8 \times 10^8 \sin^2 \beta \text{ GeV}$ (the white dwarf cooling fit is actually improved for $f_a \approx 1 \times 10^9 \sin^2 \beta$) [18, 25–27].

⁸ If inflation occurred after PQ symmetry breaking then a smaller θ can be anthropically chosen, allowing $f_a > 10^{12} \text{ GeV}$ [32].

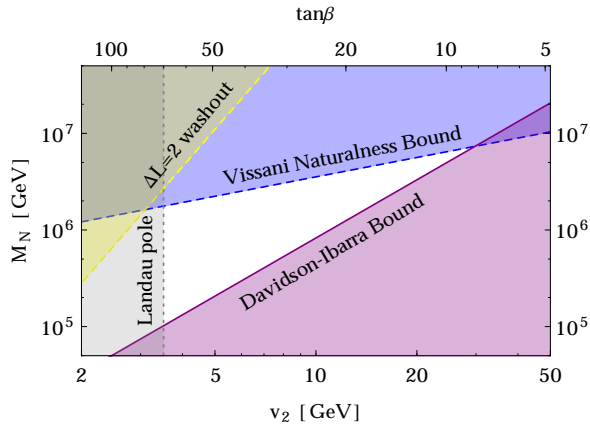


FIG. 2. Constraints on the right-handed neutrino masses. The naturalness bound on M_N corresponds to the rough bound Eq. (22) evaluated at $m_{22} = 1$ TeV.

quark scatterings. The $\Delta L = 2$ scatterings *can* however constitute a significant departure from the standard case, particularly in $K \ll 1$ scenarios dependent on initial conditions. For the parameter space of interest to us, the generated asymmetry is safe from strong $\Delta L = 2$ scattering washout, as shown in Fig. 2 [9]. As well, since leptogenesis in this model will be occurring at temperatures below 10^9 GeV, the Yukawa couplings will be in equilibrium and flavour effects cannot be ignored (see e.g. Refs. [41–45]).⁹

These departures from the standard scenario deserve further detailed study. Still, we do not expect the picture to be dramatically changed. In particular we expect the Davidson–Ibarra bound [46, 47] for successful hierarchical thermal leptogenesis, scaled for the differing vev in Eq. (10), to approximately hold:

$$M_{N_1} \gtrsim \frac{5 \times 10^8 \text{ GeV}}{\tan^2 \beta}. \quad (15)$$

This bound is depicted in Fig. 2.

Let us briefly demonstrate that this picture is consistent. A simple example configuration which achieves maximal CP violation and saturates the Davidson–Ibarra bound is (assuming normal ordering) $m_1 \ll \tilde{m}_1$ and

$$R = \begin{pmatrix} \sqrt{1 - R_{31}^2} & 0 & R_{31} \\ 0 & 1 & 0 \\ R_{31} & 0 & -\sqrt{1 - R_{31}^2} \end{pmatrix}, \quad (16)$$

⁹ An additional consideration is $N_1 N_1 \rightarrow aa$ annihilations. Ref. [7] estimates $\Gamma_{N_1 N_1 \rightarrow aa} \sim 10^{-2} M_{N_1}^5 / \langle S \rangle$ at $T = M_{N_1}$, which implies the out-of-equilibrium condition $M_{N_1} \lesssim 10^9$ GeV, easily satisfied the parameter space of interest.

¹⁰ It still may be the case that quantum gravity effects at the Planck scale induce a naturalness problem. However, this cannot be rig-

where $R_{31} \equiv \kappa \exp(i\pi/4)$. Here κ is related to the decay parameter by

$$\kappa \approx \frac{0.15\sqrt{K}}{\tan \beta} \left(\frac{0.05 \text{ eV}}{m_3} \right)^{\frac{1}{2}}, \quad (17)$$

and is typically $\lesssim 10^{-2}$ in the parameter range of interest. In the limit $m_1 = 0$, this corresponds to a $U y_\nu$ which has one zero row, but is otherwise approximately diagonal. We note that, in this configuration, \tilde{m}_1 and the CP asymmetry are sufficiently stable under radiative corrections. The reader may check this claim against the renormalisation group equation (RGE) in Appendix A.

IX. Our naturalness philosophy: A naturalness problem arises when a low mass scale is subject to large and physically meaningful quantum corrections from a high mass scale. In particular, the electroweak scale can receive such corrections from high-scale new physics.

The RGE formalism is a sensible way to quantify a naturalness problem in any perturbative quantum field theory. For example, in the SM, the $\mu^2(\mu_R)$ RGE is dominated by the top quark Yukawa,

$$\frac{d\mu^2}{d \ln \mu_R} \approx \frac{1}{(4\pi)^2} 6y_t^2 \mu^2, \quad (18)$$

where μ_R is a representative energy scale. From the RGE perspective, μ^2 is not subject to any large physical corrections and can therefore be considered natural, in the sense that once it is set to be electroweak scale it stays as such within the region of validity of the model ($\mu_R < M_{Pl}$).¹⁰ It follows intuitively that $\mu^2(\mu_R \gg m_Z)$ is not finely tuned against $\mu^2(m_Z)$. In the following we demonstrate that there exists a region of ν DFSZ parameter space where our phenomenological goals can be achieved and the heavy PQ scale induces no naturalness problem, i.e., all scales remain stable under RG evolution.

X. Naturalness in the ν DFSZ: Defining $\mathcal{D} \equiv (4\pi)^2 \frac{d}{d \ln \mu_R}$ and keeping only $y_{t,b,\tau,\nu}$ Yukawas, the one-loop RGEs for the [Type II, Flipped] ν 2HDM mass parameters can be written [51]

ously computed in the absence of a cogent theory of quantum gravity, so an agnostic stance on this possible problem seems reasonable to us. In other words, we remain agnostic to the physics which sets the boundary condition $\mu^2(\mu_R \gg m_Z) \lesssim \text{TeV}^2$ at some high scale; this is a separate problem – a hierarchy-type problem – which may or may not have a natural solution (see

$$\mathcal{D}m_{12}^2 = m_{12}^2 \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 2\lambda_3 + 4\lambda_4 + 2\lambda_S + 4\lambda_{1S} + 4\lambda_{2S} + 3y_t^2 + 3y_b^2 + y_\tau^2 + \text{Tr}(y_\nu^\dagger y_\nu) \right), \quad (19)$$

$$\mathcal{D}m_{11}^2 = m_{11}^2 \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6\lambda_1 + 6y_t^2 + [0, 2y_\tau^2] \right) + m_{22}^2 (4\lambda_3 + 2\lambda_4) + \langle S \rangle^2 \lambda_{1S} (4\lambda_{1S} + 4\lambda_S) + M_{SS}^2 2\lambda_{1S}, \quad (20)$$

$$\begin{aligned} \mathcal{D}m_{22}^2 = m_{22}^2 \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6\lambda_2 + 6y_b^2 + [2y_\tau^2, 0] + 2\text{Tr}(y_\nu^\dagger y_\nu) \right) + m_{11}^2 (4\lambda_3 + 2\lambda_4) \\ - 4\text{Tr}(y_\nu M_N^2 y_\nu^\dagger) + \langle S \rangle^2 \lambda_{2S} (4\lambda_{2S} + 4\lambda_S) + M_{SS}^2 2\lambda_{2S}, \end{aligned} \quad (21)$$

where $M_N^2 = y_N^\dagger y_N \langle S \rangle^2$ is the (diagonalised) right-handed neutrino mass matrix. The $\langle S \rangle^2$ and M_{SS}^2 terms correspond to the contribution from the heavy real singlet s in the broken phase. We provide the full list of RGEs in Appendix A.

These RGEs make manifest the decoupling limit $\epsilon, \lambda_{1S}, \lambda_{2S}, \text{Tr}(y_\nu^\dagger y_\nu y_N^\dagger y_N) \rightarrow 0$ which protects the scales from large corrections. First, corrections to m_{12}^2 are proportional to m_{12}^2 , reflecting the fact that $\epsilon \rightarrow 0$ reinstates a $U(1)_L$ symmetry. Second, because the parameters $\lambda_{3,4}$ are reintroduced by gauge loops, m_{11}^2 can only be protected from m_{22}^2 by having m_{22}^2 not too much larger than m_{11}^2 ; it was argued in Ref. [9] that $m_{22} \lesssim \text{few} \times 10^3$ GeV can accommodate a 10% fine-tuning measured at M_{Pl} . Third, m_{22}^2 is protected from the M_N by (roughly) $\text{Tr}(y_\nu^\dagger y_\nu y_N^\dagger y_N)/(4\pi^2) \lesssim m_{22}^2/\langle S \rangle^2$; this translates to the sufficient condition [52, 53]

$$M_N \lesssim \frac{3 \times 10^7 \text{ GeV}}{\tan^{\frac{2}{3}} \beta} \left(\frac{m_{22}}{\text{TeV}} \right)^{\frac{2}{3}}, \quad (22)$$

for all the right-handed neutrinos, illustrated in Fig. 2. Last, the m_{ii}^2 are protected from the PQ scale by (again roughly) $\lambda_{iS} \lesssim m_{ii}^2/\langle S \rangle^2$. We note that there is a lepton box induced correction to λ_{2S} ; this correction is also bounded by $m_{ii}^2/\langle S \rangle^2$ through Eq. (22).

XI. Explicit example: As a final demonstration we thought it illustrative to solve the coupled set of RGEs for an explicit example. We consider $\tan \beta = 30$ and neglect running in the following six quantities:

$$\begin{aligned} M_{SS}^2 &= -(10^{11} \text{ GeV})^2, & M_{N_1} &= 6 \times 10^5 \text{ GeV}, \\ \langle S \rangle^2 &= -M_{SS}^2/\lambda_S, & M_{N_2} &= M_{N_3} = 10^6 \text{ GeV}, \\ \lambda_S &= 0.26, & y_N &= M_N/\langle S \rangle. \end{aligned} \quad (23)$$

We have taken M_{N_1} at the Davidson–Ibarra bound and $M_{N_{1,2}}$ below the rough naturalness bound Eq. (22). We take y_ν according to Eqs. (11), (16) and (17) with $K = 1$,

and neglect running here as well. A glance at the RGEs in Appendix A will convince the reader that neglecting running in these parameters is a good approximation. Decoupling of the heavy degrees of freedom at m_s, M_{N_i} , and m_{22} should be accompanied by an associated shift in the λ_j parameters, matching to the effective theory below each threshold. However, in practice, since the λ_{iS}, y_ν are so small and m_{22} is not too much larger than m_Z , it makes little numerical difference to implement this shift. Therefore we evolve the following parameters under the ν DFSZ RGEs:¹¹

$$\begin{aligned} \lambda_3(m_{22}) &= \lambda_4(m_{22}) = 0.02, & \lambda_{1S}(m_s) &= 10^{-18}, \\ \lambda_1(m_Z) &= \lambda_2(m_Z) = 0.26 & \lambda_{2S}(m_s) &= 10^{-16}, \\ y_t(m_Z) &= 0.96/\sin \beta, & g_1^2(m_Z) &= 0.13, \\ y_b(m_Z) &= 0.017/\cos \beta, & g_2^2(m_Z) &= 0.43, \\ y_\tau(m_Z) &= 0.010/\cos \beta, & g_3^2(m_Z) &= 1.48. \end{aligned}$$

Their evolution is shown in Appendix B.¹²

To evolve the mass parameters we set $m_{11}^2(m_{22}) = -(88 \text{ GeV})^2$ and consider $m_{22}(m_{22}) = 0.6, 0.8, 1.0, 2.0$ TeV. The N_i and s are decoupled with step functions at their thresholds. Their RG evolution is shown in Fig. 3; it is plain that the mass parameters in this (viable) example remain relatively small up to high scales, and are therefore natural according to our philosophy.

XII. Conclusion : We have described an extension of the SM (the “ ν DFSZ”) by three right-handed neutrinos, a complex scalar doublet, and a complex scalar singlet. The ν DFSZ serves as an existence proof that weakly coupled high-scale physics can explain phenomenological shortcomings of the SM *without introducing a naturalness problem*. The model explains neutrino masses, the BAU, the strong CP problem, and dark matter, via a seesaw mechanism, hierarchical leptogenesis, the PQ

e.g. Refs. [48–50] for relevant discussion). The naturalness question induced by the high PQ scale can, by contrast, be fully analysed within quantum field theory, and we limit our scope to that.

¹¹ For definiteness we take a Type II arrangement, but the Flipped arrangement gives very similar results.

¹² We note that the parameter λ_1 tends to run negative, threatening the stability of the electroweak vacuum; nevertheless we expect the problem to be no worse than in the SM, i.e., we expect a metastable vacuum.

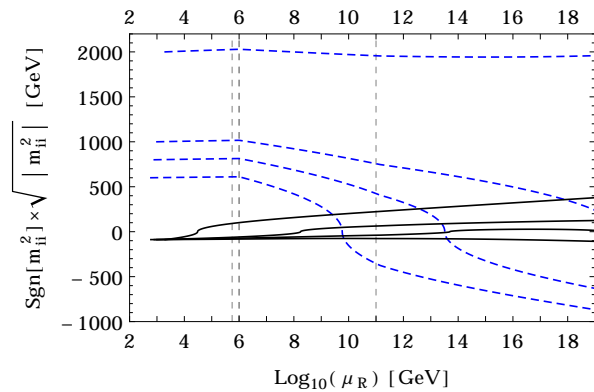


FIG. 3. Example RG evolution (see text) of $m_{11}^2(\mu_R)$ (black solid) and $m_{22}^2(\mu_R)$ (blue dashed) for $m_{22} = 0.6, 0.8, 1.0, 2.0$ TeV bottom to top.

mechanism, and a DFSZ invisible axion, respectively. It contains four scales: $|m_{11}| \approx 88$ GeV, $m_{22} \sim 10^3$ GeV, $M_N \sim 10^5\text{--}10^7$ GeV, and $M_{PQ} \sim 10^{11}$ GeV, each protected from quantum corrections by a technically natural decoupling limit. The \sim TeV-scale scalars and the invisible axion of the model will be probed in upcoming experiments.

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A. RGEs : Following is the full list of RGEs in the [Type II, Flipped] model, found using PyR@TE [51]. Shown in blue/underlined are those parameters which, for simplicity, we did not evolve in our RGE analysis.

$$\mathcal{DM}_{11}^2 = M_{11}^2 \left(6\lambda_1 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6y_t^2 + [0, 2y_\tau^2] \right) + M_{22}^2 (4\lambda_3 + 2\lambda_4) + M_{SS}^2 2\lambda_{1S}, \quad (\text{A1})$$

$$\mathcal{DM}_{22}^2 = M_{22}^2 \left(6\lambda_2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6y_b^2 + [2y_\tau^2, 0] + 2\text{Tr} (y_\nu^\dagger y_\nu) \right) + M_{11}^2 (4\lambda_3 + 2\lambda_4) + M_{SS}^2 2\lambda_{2S}, \quad (\text{A2})$$

$$\underline{\mathcal{DM}}_{SS}^2 = M_{SS}^2 \left(4\lambda_S + \text{Tr} (y_N^\dagger y_N) \right) + M_{11}^2 4\lambda_{1S} + M_{22}^2 4\lambda_{2S}, \quad (\text{A3})$$

$$\underline{\mathcal{D}\langle S \rangle^2} = -\text{Tr} (y_N^\dagger y_N) \langle S \rangle^2 \quad [\text{i.e. the wave function renormalisation}], \quad (\text{A4})$$

$$\mathcal{D}g_{\{1,2,3\}} = \{7, -3, -7\} g_{\{1,2,3\}}^3, \quad (\text{A5})$$

$$\begin{aligned} \mathcal{D}\lambda_1 = & \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4 - \lambda_1 (3g_1^2 + 9g_2^2) + 12\lambda_1^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_{1S}^2 \\ & + 12\lambda_1y_t^2 - 12y_t^4 + [0, 4\lambda_1y_\tau^2 - 4y_\tau^4], \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} \mathcal{D}\lambda_2 = & \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4 - \lambda_2 (3g_1^2 + 9g_2^2) + 12\lambda_2^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_{2S}^2 \\ & + 12\lambda_1y_b^2 - 12y_b^4 + [4\lambda_1y_\tau^2 - 4y_\tau^4, 0] + 4\lambda_2\text{Tr} (y_\nu^\dagger y_\nu) - 4\text{Tr} (y_\nu^\dagger y_\nu y_\nu^\dagger y_\nu), \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \mathcal{D}\lambda_3 = & \frac{3}{4}g_1^4 - \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4 - \lambda_3 (3g_1^2 + 9g_2^2) + (6\lambda_3 + 2\lambda_4) (\lambda_1 + \lambda_2) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_{1S}\lambda_{2S} \\ & + \lambda_3 (6y_t^2 + 6y_b^2 + 2y_\tau^2 + 2\text{Tr} (y_\nu^\dagger y_\nu)) - 12y_t^2y_b^2 - [0, 4 (y_\nu^\dagger y_\nu)_{33} y_\tau^2], \end{aligned} \quad (\text{A8})$$

$$\begin{aligned} \mathcal{D}\lambda_4 = & 3g_1^2g_2^2 - \lambda_4 (3g_1^2 + 9g_2^2) + 2\lambda_4 (\lambda_1 + \lambda_2) + 8\lambda_4\lambda_3 + 4\lambda_4^2 \\ & + \lambda_4 (6y_t^2 + 6y_b^2 + 2y_\tau^2 + 2\text{Tr} (y_\nu^\dagger y_\nu)) + 12y_t^2y_b^2 + [0, 4 (y_\nu^\dagger y_\nu)_{33} y_\tau^2], \end{aligned} \quad (\text{A9})$$

$$\underline{\mathcal{D}\lambda_S} = 10\lambda_S^2 + 2\lambda_S\text{Tr} (y_N^\dagger y_N) + 4\lambda_{1S}^2 + 4\lambda_{2S}^2 - 2\text{Tr} (y_N^\dagger y_N y_N^\dagger y_N), \quad (\text{A10})$$

$$\begin{aligned} \mathcal{D}\lambda_{1S} = & \lambda_{1S} \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 4\lambda_{1S} + 4\lambda_S + 6\lambda_1 \right) + \lambda_{2S} (4\lambda_3 + 2\lambda_4) \\ & + \lambda_{1S} \left(6y_t^2 + [0, 2y_\tau^2] + \text{Tr} (y_N^\dagger y_N) \right), \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} \mathcal{D}\lambda_{2S} = & \lambda_{2S} \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 4\lambda_{2S} + 4\lambda_S + 6\lambda_2 \right) + \lambda_{1S} (4\lambda_3 + 2\lambda_4) \\ & + \lambda_{2S} \left(6y_b^2 + [2y_\tau^2, 0] + 2\text{Tr} (y_\nu^\dagger y_\nu) + \text{Tr} (y_N^\dagger y_N) \right) - 4\text{Tr} (y_\nu^\dagger y_\nu y_N^\dagger y_N), \end{aligned} \quad (\text{A12})$$

$$\mathcal{D}\epsilon = \epsilon \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 2\lambda_3 + 4\lambda_4 + 2\lambda_S + 4\lambda_{1S} + 4\lambda_{2S} + 3y_t^2 + 3y_b^2 + y_\tau^2 + \text{Tr} (y_\nu^\dagger y_\nu) + \text{Tr} (y_N^\dagger y_N) \right), \quad (\text{A13})$$

$$\mathcal{D}y_t = y_t \left(-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{9}{2}y_t^2 + \frac{1}{2}y_b^2 + [0, y_\tau^2] \right), \quad (\text{A14})$$

$$\mathcal{D}y_b = y_b \left(-\frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{9}{2}y_b^2 + \frac{1}{2}y_t^2 + [y_\tau^2, 0] \right), \quad (\text{A15})$$

$$\mathcal{D}y_\tau = y_\tau \left(-\frac{15}{4}g_1^2 - \frac{9}{4}g_2^2 + \frac{5}{2}y_\tau^2 + [3y_b^2, 3y_t^2] \right), \quad (\text{A16})$$

$$\underline{\mathcal{D}y_\nu} = y_\nu \left(-\frac{3}{4}g_1^2 - \frac{9}{4}g_2^2 + 3y_b^2 + \text{Tr} (y_\nu^\dagger y_\nu) \right) + \left[y_\nu y_\tau^2 - \frac{3}{2}\text{Diag} (0, 0, y_\tau^2) y_\nu, 0 \right] + \frac{3}{2}y_\nu y_\nu^\dagger y_\nu + \frac{1}{2}y_\nu y_N^\dagger y_N, \quad (\text{A17})$$

$$\underline{\mathcal{D}y_N} = \frac{1}{2}\text{Tr} (y_N^\dagger y_N) y_N + y_N y_N^\dagger y_N + y_N y_\nu^\dagger y_\nu + y_\nu^T y_\nu^* y_N. \quad (\text{A18})$$

B. Explicit example RG evolution : Shown below is the RG evolution of dimensionless parameters in our explicit example (as a function of $\log_{10} \mu_R$).

